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RESEARCH ARTICLE

Qualitative Analysis of a System-Integrated Organizational and Technological Differential Model of Energy-Efficient Construction

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Abstract

Energy-efficient construction projects are complex organizational and technological systems in which costs, revenues, labor, equipment, and energy resources interact dynamically. Traditional static or linear models are often insufficient for describing nonlinear changes in project performance and for identifying critical parameter combinations that may alter the behavior of the system. This study aimed to develop and qualitatively analyze a system-integrated organizational and technological differential model for energy-efficient construction, with particular attention to the dynamics of costs and revenues during project implementation. The proposed model was formulated as a system of autonomous differential equations incorporating key resource groups, including labor, equipment, energy consumption, and material inputs. The research applied methods of mathematical modeling, qualitative analysis of differential systems, Lyapunov stability theory, phase-space analysis, and bifurcation analysis. Numerical experiments were conducted using representative parameter values, and the system behavior was visualized through phase portraits and bifurcation diagrams. The analysis made it possible to identify equilibrium points of the system, determine their stability characteristics, and describe several possible behavioral regimes under different parameter configurations. The results show that combinations of model parameters, rather than isolated values, play a decisive role in the transition between stable and unstable system states. The obtained findings demonstrate the practical relevance of nonlinear dynamic modeling for construction management, especially in situations where resource constraints and cost-revenue interactions must be considered simultaneously. **Conclusions:** The proposed differential model provides a useful analytical tool for assessing, forecasting, and optimizing organizational and technological processes in energy-efficient construction projects.

Keywords: energy-efficient construction; differential model; system analysis; phase space; system stability; bifurcation analysis.

1 Introduction

The relevance of this study is determined by the fact that complex systems form the basis for the functioning of most economic processes in general and the construction industry in particular. At the same time, there is a need to develop effective approaches for managing such systems in order to achieve the desired economic or social outcome. The description of real systems is a challenging scientific task because universal modeling methods do not exist; for each type of model, specific approaches must therefore be developed and justified.

In practice, research is often limited to the use of simplified, predominantly linear models and methods. Such models may be useful at the initial stage because they allow researchers to form a general understanding of the system and identify the basic relationships among its parameters. However, it is obvious that a comprehensive study of complex systems is impossible without taking into account nonlinear effects, which significantly influence their behavior.

In particular, the analysis of the bifurcation properties of systems is of great importance – namely, the ability of a system to undergo qualitative changes in behavior under smooth variation of parameters. Of particular interest are

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degenerate systems, in which certain parameters reach critical or limiting values. While bifurcation studies are sometimes carried out for nondegenerate (robust) systems, the corresponding approaches for degenerate systems remain practically undeveloped.

As is known, the system-integrated organizational and technological model of energy-efficient construction (SIOTM) makes it possible to estimate costs during the construction process using meaningful information about the applied process [1, 2]. The large number of SIOTM parameters allows the model to be treated as comparatively stable and resistant to random errors associated with variations in input data, mathematical modeling procedures, and numerical experiments. However, SIOTM primarily makes it possible to estimate costs rather than investigate their dynamic evolution and, consequently, does not directly support analysis of how the final result changes when input data and model parameters are varied. Considering the necessity of studying the system in dynamics and the possibility of controlling the final state of the system by varying the initial conditions and its parameters, a differential model will be used. Taking into account that the cost process in construction does not directly depend on the current time, the system of differential equations proposed as a mathematical model will be considered in an autonomous form [3].

Accurate and complete representation (modeling) of real systems is a complex scientific task, since universal modeling schemes do not exist – specific approaches must be developed for each class of models. Quite often, research in this field is limited to the use of simplified, mainly linear, models and the corresponding analytical tools. Such an approach is justified at the initial stage, as it allows the determination of the general structure of the system and the establishment of the main relationships between its parameters.

At the same time, the analysis of complex systems cannot be reduced exclusively to linear schemes without taking into account nonlinear effects, which significantly influence their functioning and accuracy. This is especially relevant to the study of bifurcation properties, namely the ability of a system to undergo qualitative changes in behavioral regimes under continuous variation of parameters.

Of particular interest are degenerate systems in which some parameters acquire critical or limiting values. While bifurcation analysis is sometimes carried out for nondegenerate (robust) systems, the corresponding methodological framework for degenerate systems remains insufficiently developed.

It should be emphasized that the formation of the system approach to the study of complex organized processes and systems took place in close interrelation with the development of the general systems theory and cybernetics, comprehensively presented in the works of L. Bertalanffy [4]. Modern mathematical modeling is mainly aimed at refining already existing models, deepening them, and improving methods of numerical and qualitative analysis. In various applied fields, their own classes of models have been formed, which are usually described by systems of differential, difference, or regression equations. Each of these directions has a well-developed theoretical foundation.

An important prerequisite for the active use of mathematical modeling in the study of complex organized processes was the dissemination of applied mathematical methods, the rapid growth of the computational capabilities of computer technology, and the expansion of the range of software tools. The main stages in the development of computer-based modeling methods and corresponding software tools are reflected in the works of H. Bossel [5] and A. Starfield [6].

The analysis of bifurcation parameter values is, in most cases, an integral component of nonlinear model research and makes it possible to determine the qualitative stages of system evolution. Numerous fundamental works and monographs by M. Braun [7], P. Hartman [8], D.W. Jordan, and P. Smith [9], among others, are devoted to the theoretical foundations of the qualitative analysis of systems of differential equations. In particular, these works address the construction of new models, their investigation using classical methods, the analysis of bifurcation properties, as well as problems of control, optimization, and other applied aspects.

To formalize the system in the form of a differential model represented by a system of autonomous differential equations, the following assumptions were made. The construction process consists in the implementation of a certain number of sequential or parallel works (manufacturing of structural elements), each of which is characterized by a specific type of material and technological process. It is assumed that different types of resources may be involved in carrying out the works (manufacturing structural elements), in particular labor, machinery, or energy.

Although the problem is inherently hybrid in nature, combining discrete and continuous features, the use of a continuous representation in the form of a system of autonomous differential equations is considered appropriate for the present analytical purpose.

The object of the study comprises mathematical models represented by systems of autonomous differential equations, together with the system invariants and the key factors that determine their behavior.

The subject of the study is the project implementation process, which consists of construction works that differ in the materials used, equipment requirements, resource intensities, and execution capacities.

The aim of the work is to develop a differential system-integrated organizational and technological model of energy-efficient construction and to investigate its stability and bifurcation properties.

The research methods include qualitative approaches to the study of systems of differential equations, mathematical modeling, systems analysis, and bifurcation analysis.

The problem statement was formulated through the following research tasks.

Within the overall objective, the following tasks were set:

- to develop a differential system-integrated organizational and technological model of energy-efficient construction;
- to identify the model parameters;
- to determine the stationary (equilibrium) points of the model;
- to investigate the stability of the system in the vicinity of singular points;
- to analyze the bifurcation properties of the model;
- to provide a practical interpretation of the obtained results.

2 Materials and methods

2.1 Mathematical model

The model is based on the concept of using three types of resources: R – labor; T – equipment; E – energy in the execution of N tasks. The proposed mathematical model takes the following form:

$$\begin{cases} \dfrac{\partial V}{\partial t} = \sum\limits_{k=1}^{N} \left(V \cdot Rr_k \cdot s_{Rr} + V^2 \cdot \sum\limits_{j=1}^{T} \left(Rt_{kj} \cdot s_{Rt} \right) + V \cdot Re_k \cdot s_{Re} + \sum\limits_{i=1}^{M} \left(Sm_i \cdot m_{ik} \right) \right), \\ \dfrac{\partial P}{\partial t} = \sum\limits_{k=1}^{N} P_k, \end{cases} \tag{1}$$

where V denotes the current (operating) expenses of the enterprise over the specified period; P denotes the current revenue of the enterprise over the specified period; N is the number of tasks (operations) defined by the technical assignment; Rr_k is the number of workers simultaneously involved in performing the k-th task; s_{Rr} is the cost of engaging a worker per unit of time (hour/day); Rt_k is the number of machines (equipment units) simultaneously involved in performing the k-th task; s_{Rt} is the cost of using equipment per unit of time (hour/day); T is the total number of equipment types used to complete the technical assignment; Re_k is the amount of energy required to perform the k-th task; s_{Re} is the cost per unit of accounted energy; M is the total number of materials used in project execution; m_{ik} is the quantity of the i-th material used for performing the k-th task; Sm_i is the cost per unit of the i-th material; and P_k is the revenue obtained from performing the k-th task.

We assume that the sequence of projects and the tasks within them is continuous and steady. In this case, it is appropriate to investigate the dynamics of costs and revenues arising from such activities.

For system (1), the following constraints are applied:

$$\sum_{k=1}^{N} Rr_k \le RR; \qquad \sum_{k=1}^{N} Rt_k \le RT; \qquad \sum_{k=1}^{N} Re_k \le RE, \tag{2}$$

where RR – total number of workers in the project team; RT – total number of equipment units involved in the project; RE – constraint on energy consumption within the project.

The mathematical framework and methodology for studying model (1)–(2) are based on a combined approach that includes qualitative system analysis and bifurcation analysis in the sense of Lyapunov, and involve the following:

i. identification of the system parameters (1)–(2);
ii. analysis of the phase space through the construction of phase trajectories under different initial conditions of the system;
iii. identification of the system equilibrium points;
iv. determination of the type of the identified equilibrium points;
v. derivation of analytical conditions for the stability of system (1)–(2) in the neighborhood of the equilibrium points;

analysis of the bifurcation properties of the system in the vicinity of the equilibrium points.

2.2 Study design

This study was designed as a theoretical and computational investigation of a system-integrated organizational and technological differential model for energy-efficient construction. The research was based on mathematical modeling, qualitative analysis of autonomous differential equations, phase-space analysis, Lyapunov stability theory, and bifurcation analysis. The purpose of the methodological approach was to describe the dynamic interaction between construction costs and revenues under the influence of different resource groups, including labor, equipment, energy, and materials.

The construction process was considered as a complex organizational and technological system consisting of a set of tasks performed sequentially or in parallel. Each task was associated with specific resource requirements and economic

indicators. Since the dynamics of the modeled system do not explicitly depend on time, the model was formulated as a system of autonomous differential equations.

2.3 Model parameters and assumptions

The study did not involve physical materials, laboratory samples, or human participants. The "materials" of the research consisted of mathematical variables, model parameters, and representative numerical data used for computational experiments.

The model included the following main groups of parameters: current operating expenses, current revenue, number of construction tasks, number of workers involved in task execution, cost of labor per unit of time, number of equipment units, cost of equipment use, amount of energy required for task performance, cost of energy, quantity of materials used, cost of materials, and revenue obtained from each task.

Resource constraints were introduced for the total number of workers, total number of equipment units, and total energy consumption within the construction project. These constraints made it possible to reflect the practical limitations that usually arise during the implementation of energy-efficient construction projects.

2.4 Mathematical modeling and analysis

At the first stage, the construction process was formalized as a system of autonomous differential equations describing the dynamics of costs and revenues. The model was based on the assumption that construction tasks are performed continuously and steadily, which allows the use of a continuous dynamic representation despite the hybrid nature of real construction processes.

At the second stage, the model parameters were identified and assigned representative numerical values. These values were used to solve the Cauchy problem for the proposed system under specified initial conditions. The obtained numerical solutions made it possible to analyze the behavior of expenses, revenues, and their combined dynamics over time.

At the third stage, the phase space of the system was investigated. Phase trajectories were constructed under different initial conditions in order to demonstrate possible behavioral regimes of the system. This allowed the identification of stable, unstable, and transitional patterns in the relationship between costs and revenues.

2.5 Stability and bifurcation analysis

The stability analysis was carried out using the Lyapunov approach. Equilibrium points were identified by setting the right-hand sides of the system of differential equations equal to zero. The system was then linearized in the neighborhood of the equilibrium point by constructing the matrix of partial derivatives.

The type of equilibrium point was determined on the basis of the eigenvalues of the corresponding characteristic matrix. Special attention was paid to degenerate cases, in which the characteristic equation has a simplified structure and bifurcation transitions may occur when specific parameter combinations reach critical values.

Bifurcation analysis was performed to determine how changes in parameter combinations influence the qualitative behavior of the system. Bifurcation diagrams were constructed to visualize the transition between different system states, including unstable nodes, saddle points, and saddle-node configurations. The analysis focused not only on individual parameter values but also on their combinations, since the interaction between parameters plays a decisive role in the dynamics of construction processes.

2.6 Data analysis and visualization

The numerical results were analyzed using qualitative and graphical methods. Phase portraits were used to visualize trajectories of the system in the phase space, while bifurcation diagrams were used to demonstrate changes in system topology under different parameter configurations. The interpretation of the results was based on the comparison of modeled dynamic regimes and their practical meaning for construction project management.

No statistical hypothesis testing was performed, as the study was based on mathematical modeling rather than empirical data collection. The analysis focused on the structural properties of the proposed model, the stability of equilibrium states, and the practical interpretation of dynamic regimes.

2.7 Ethical considerations

This study did not involve human participants, animal subjects, personal data, or confidential project documentation. Therefore, ethical approval was not required. All numerical values used in the computational experiment were representative model parameters applied for illustrative and analytical purposes.

3 Results and discussion

3.1 Numerical results and phase-space behavior

Parameter identification.. For the numerical experiment, the model parameters are assigned representative (assumed) values in accordance with the following specifications:

$V_0 = 0$; $P_0 = 0$; $N = 3$; $Rr_1 = 2$; $Rr_2 = 3$; $Rr_3 = 2$; $s_{Rr} = 200$ UAH per hour; $Rt_1 = 2$; $Rt_2 = 3$; $Rt_3 = 1$; $s_{Rt} = 700$ UAH per hour; $T = 5$; $Re_1 = 20$; $Re_2 = 30$; $Re_3 = 15$; $s_{Re} = 10$; $M = 2$; $m_{11} = 20$; $m_{12} = 15$; $m_{13} = 17$; $m_{21} = 23$; $m_{22} = 18$; $m_{23} = 29$; $Sm_1 = 120$; $Sm_2 = 85$; $P_1 = 500$; $P_2 = 700$; $P_3 = 600$; $RR = 3000$; $RT = 2000$; $RE = 1700$.

Based on the parameter values and the initial conditions V0 and P0, the Cauchy problem for system (1) with constraints (2) and the initial conditions/parameters specified above was solved. The results of solving the Cauchy problem are presented in Figure 1.

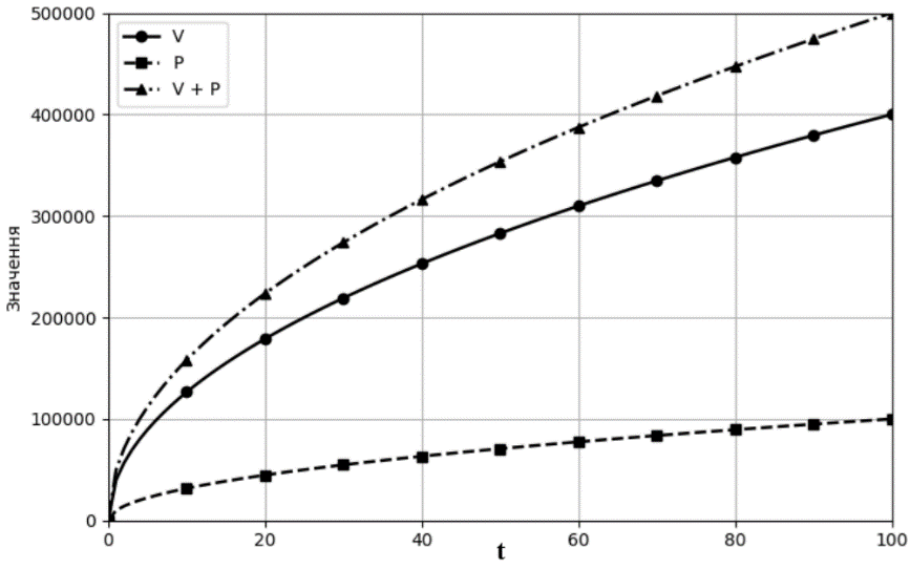


Figure 1. Results of solving the Cauchy problem for system (1) with constraints (2) and the specified initial conditions/parameters

Study of the phase space.. Figure 1 presents the results of solving the Cauchy problem for system (1) under the specified initial conditions and parameters. Such a result may be compared with the results obtained using SIOTM, that is, with the static modeling results. The modeling results may be considered satisfactory or unsatisfactory, and at the stage of phase space investigation, the problem of controlling the final state of the system under variation of the initial conditions may be formulated.

Let us consider the initial conditions in the form of (3).

$$\begin{aligned} V_0 &= [-10; -8; \ldots; -2; 0; 2; \ldots; 8; 10], \\ P_0 &= [-10; -8; \ldots; -2; 0; 2; \ldots; 8; 10]. \end{aligned} \tag{3}$$

The phase portrait of system (1)–(2) with the initial conditions (3) and other parameters specified above is presented in Figure 2.

It should be noted that, depending on the problem conditions, the parameters may vary, and for a model containing about 30 parameters, it is important not so much their individual values as the combinations of such parameters. Bifurcation transitions occur when the parameter combinations pass through zero values, and for 30 parameters there exists a sufficiently large number of such combinations. Therefore, in order to demonstrate the spectrum of possible system topologies, Figure 3 presents several variants of the phase space topology of system (1) with constraints (2).

It should be noted that for certain combinations of the parameters of model (1) with constraints (2), an additional 16 topologies are possible, of which the hyperbolic ones are shown in Figure 4, and the elliptic ones in Figure 5.

It should be noted that, in practice, the parameter combinations that give rise to the topologies shown in Figures 4–5 may occur; however, there is no applied interpretation that would qualitatively distinguish them from simpler topologies in practical problems.

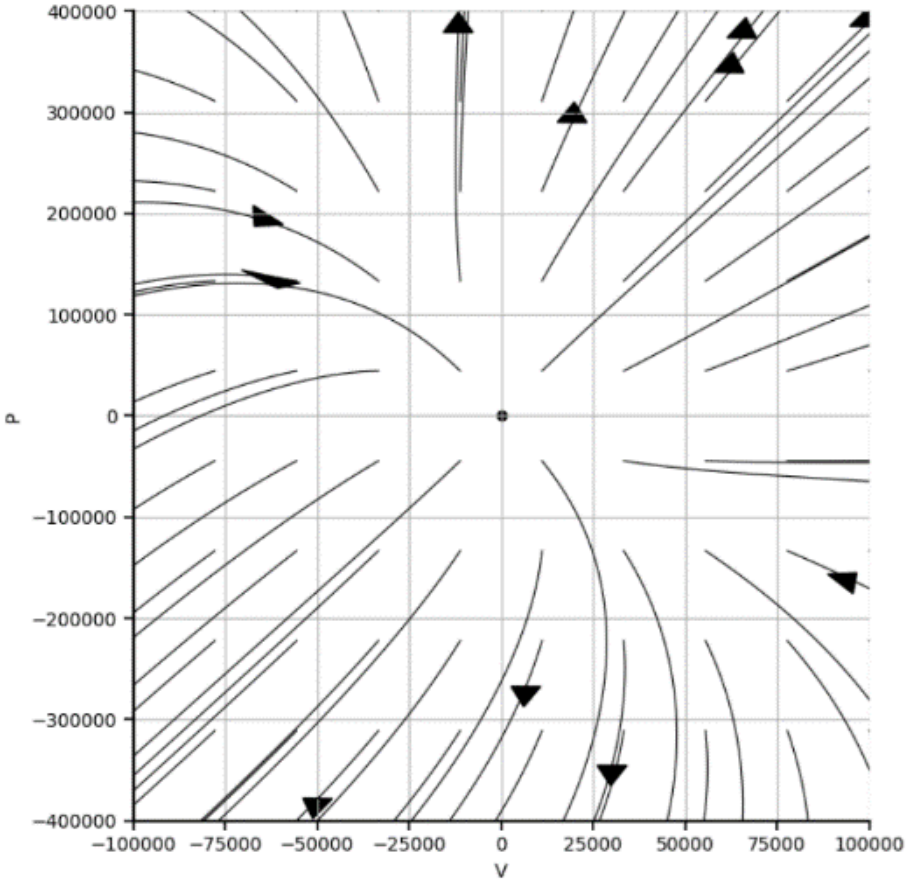


Figure 2. Phase portrait of system (1)–(2) with the parameters specified above and the initial conditions $V_0 = [-100000, -80000, \ldots, 0, \ldots, 80000, 100000]$, $P_0 = [-400000, -320000, \ldots, 0, \ldots, 320000, 400000]$

Identification of the system's equilibrium points.. By applying the methodology of Aleksandr Lyapunov stability analysis, namely by setting the right-hand side of system (1) equal to zero, we obtain:

$$\sum_{k=1}^{N}\left(V \cdot Rr_k \cdot s_{Rr} + V^2 \cdot \sum_{j=1}^{T}(Rt_{kj} \cdot s_{Rt}) + V \cdot Re_k \cdot s_{Re} + \sum_{i=1}^{M}(Sm_i \cdot m_{ik})\right) = 0,$$

$$\sum_{k=1}^{N} P_k = 0$$

It is easy to see that the system has a trivial solution; that is, the equilibrium point of the system is the origin.

Stability analysis of the system.. To study the stability of the equilibrium point, we linearize the system in the neighborhood of this point by computing the matrix of partial derivatives presented in the general form (4).

$$J_{ij} = \frac{\partial f_i(\overline{x})}{\partial x_j}. \tag{4}$$

Matrix (4) of system (1) with constraints (2) takes the form (5).

$$\begin{pmatrix} \sum_{k=1}^{N}\left(Rr_k \cdot s_{Rr} + 2V \cdot \sum_{j=1}^{T}(Rt_{kj} \cdot s_{Rt}) + Re_k \cdot s_{Re} + \sum_{i=1}^{M}(Sm_i \cdot m_{ik})\right) & 0 \\ 0 & 1 \end{pmatrix}. \tag{5}$$

As can be seen from (5), the secondary diagonal of the characteristic matrix consists of zeros. This means that the characteristic equation of system (1) with constraints (2) has a degenerate form. In general, the characteristic equation

of a system of two differential equations has the form: $\lambda^2 - \lambda \cdot Tr(J) + Det(J) = 0$ and in this (general case), bifurcations in the system arise under the conditions:

$$\begin{cases} Det(J) = 0, \\ Tr(J) = 0, \\ Tr^2(J) - 4Det(J) = 0. \end{cases} \tag{6}$$

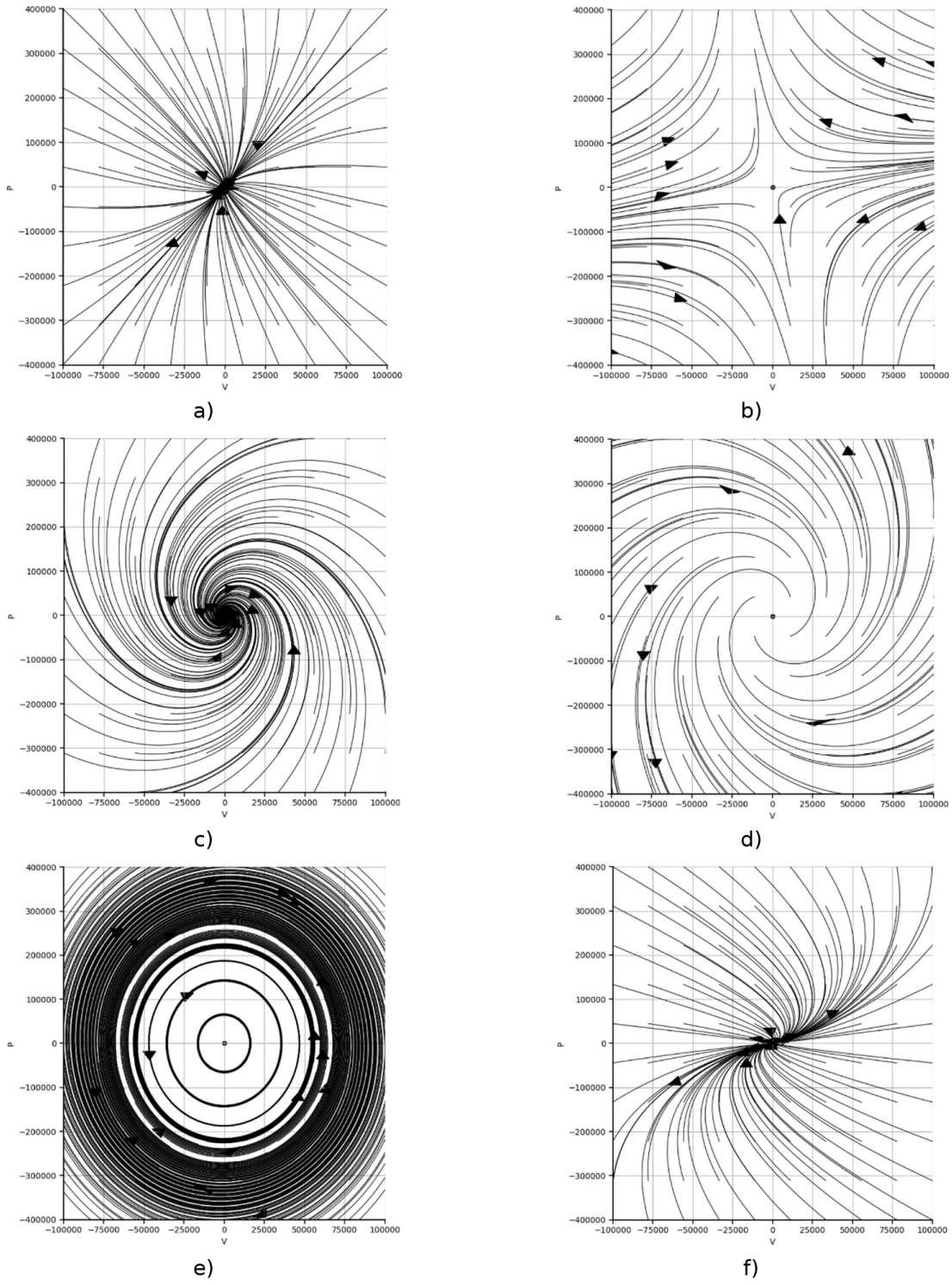


Figure 3. Phase portraits of model (1) with constraints (2) for different values of the model parameters and their combinations, corresponding to the following topologies: (a) stable node, (b) saddle, (c) stable focus, (d) unstable focus, (e) center, (f) dicritical node

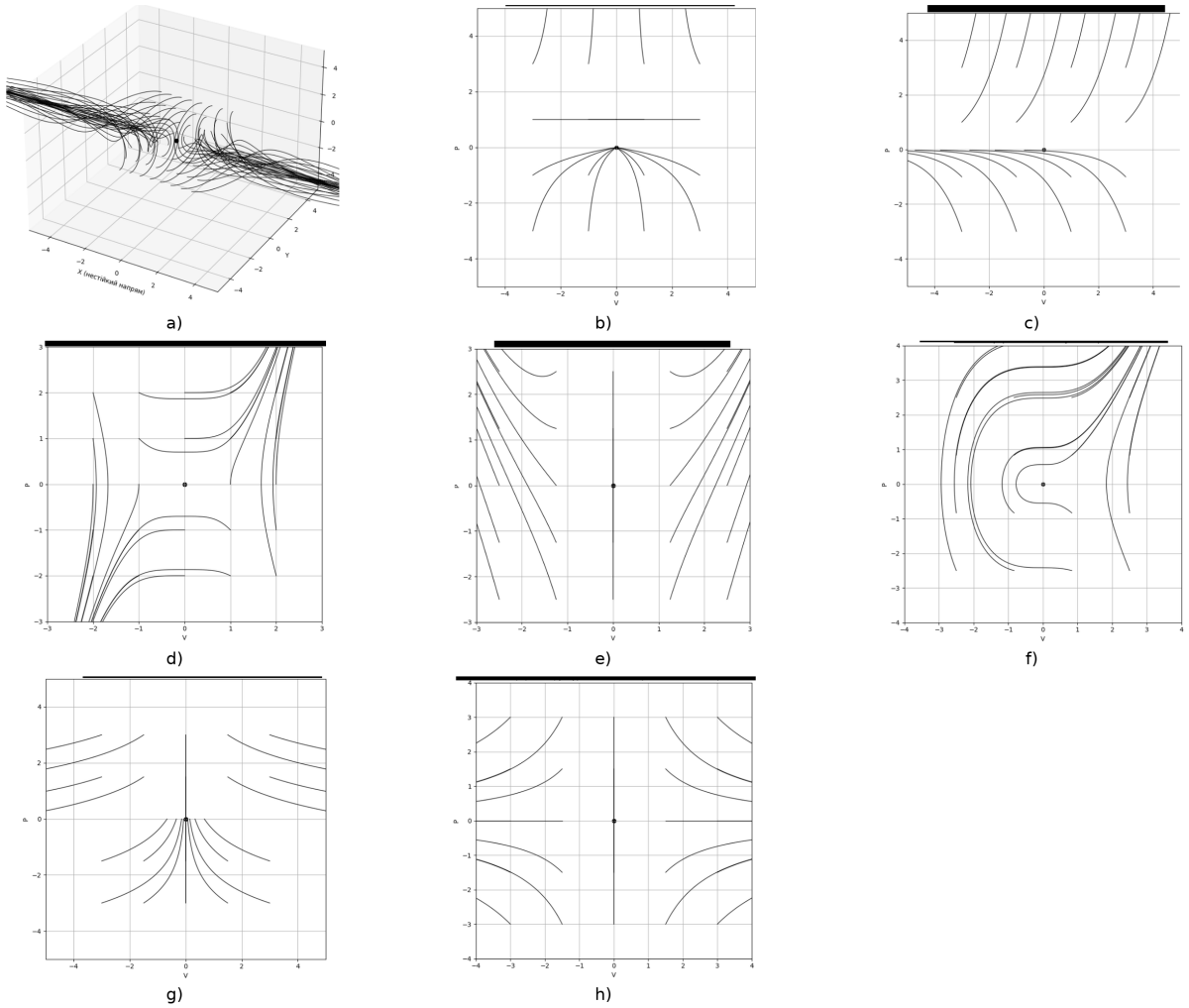


Figure 4. Possible hyperbolic topologies of the phase space of system (1) with constraints (2): (a) saddle-focus, (b) semi-stable node, (c) parabolic point, (d) cusp point, (e) degenerate saddle, (f) saddle-node, (g) transcritical point, (h) structurally stable hyperbolic state

If the elements of the secondary diagonal of the characteristic matrix are equal to zero, then the eigenvalues of the characteristic matrix are the elements of the main diagonal. Based on the above, two conclusions can be drawn:

i. the eigenvalues of the characteristic matrix are real;
ii. the equilibrium point at the origin of system (1) with constraints (2) is hyperbolic.

Analysis of the parameter space.. As shown above, the characteristic equation of the characteristic matrix of system (1) with constraints (2) has a degenerate structure; therefore, the analytical conditions for the system's stability can be identified immediately. For convenience, let us denote expression (7).

$$\sum_{k=1}^{N}\left(Rr_k \cdot s_{Rr} + 2\,V \cdot \sum_{j=1}^{T}(Rt_{kj} \cdot s_{Rt}) + Re_k \cdot s_{Re} + \sum_{i=1}^{M}(Sm_i \cdot m_{ik})\right). \tag{7}$$

i. the equilibrium point is an unstable node if (7) > 0;
ii. the equilibrium point is a saddle if (7) < 0;
iii. the equilibrium point is a saddle-node if (7) $= 0$.

For better understanding and visualization of the dependence of the type of the equilibrium point of system (1) with constraints (2), a bifurcation diagram was constructed, which is shown in Figure 6.

The results presented in Figure 6 are sufficiently logical and predictable, have a clear applied interpretation, and correspond to the applied formulation of the problem. It should be noted that the range of the presented parameter spaces makes it possible to assess the broad capabilities of system (1) with constraints (2) for various types of behavior, despite the relatively simple structure of the model.

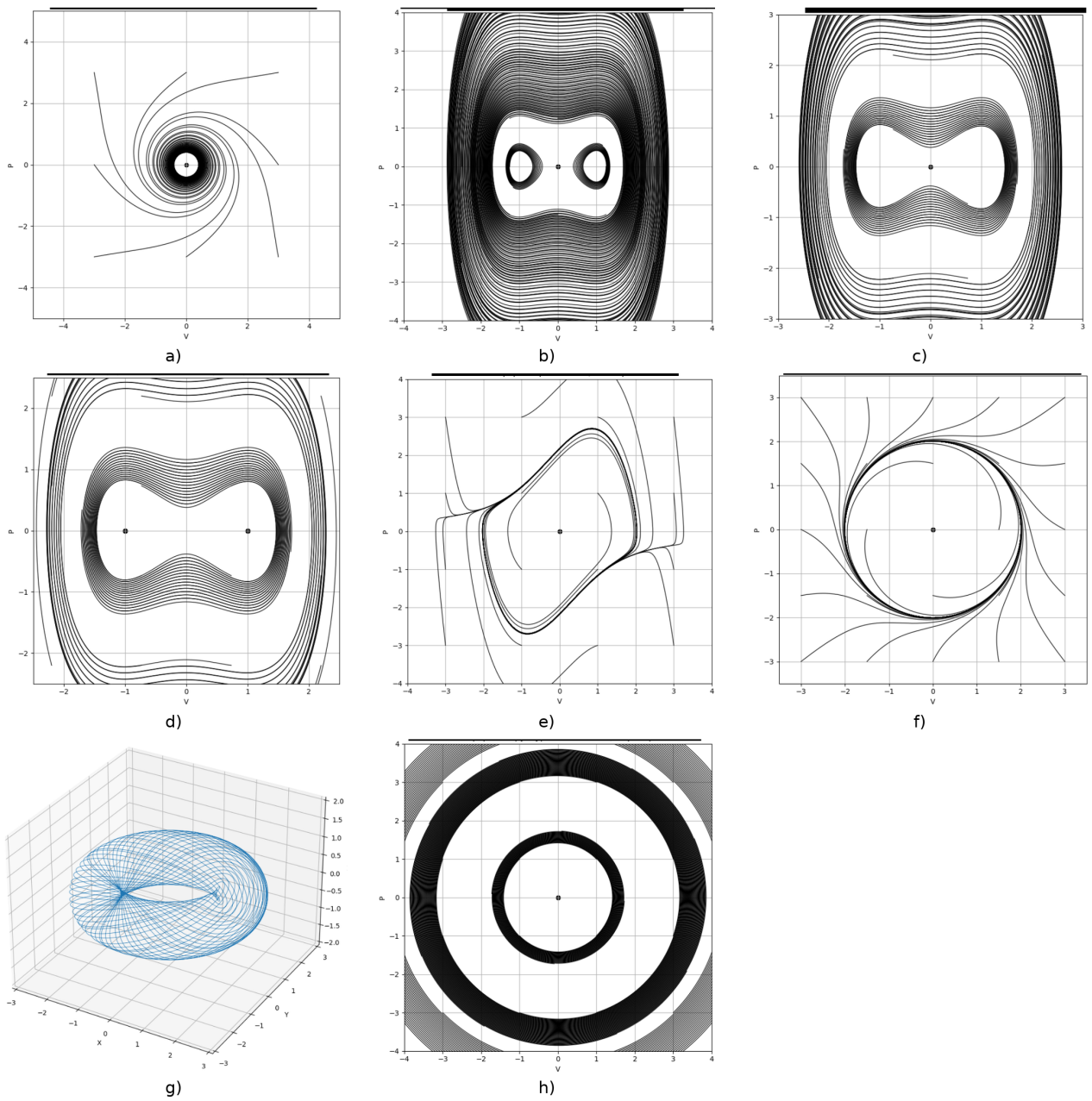


Figure 5. Possible elliptic topologies of the phase space of system (1) with constraints (2):(a) center–focus, (b) pitchfork bifurcation point, (c) homoclinic loop, (d) heteroclinic trajectory, (e) limit cycle, (f) double limit cycle, (g) double limit cycle, (h) structurally unstable center

3.2 Interpretation and practical implications

The obtained results demonstrate the importance of applying nonlinear dynamic modeling to the analysis of energy-efficient construction projects. Traditional organizational and technological models usually describe construction processes in a static form and are mainly focused on estimating costs at a specific stage of project implementation. However, such models do not sufficiently reflect the dynamic relationship between costs, revenues, and resource constraints. The proposed differential model addresses this limitation by allowing the system to be studied over time.

The results of the phase-space analysis indicate that energy-efficient construction projects should be considered as complex systems whose behavior may change significantly under different parameter configurations. This is particularly important for construction management, since small changes in resource distribution may lead to substantial changes in the economic trajectory of the project. For example, excessive use of labor or equipment may increase operating expenses and shift the system toward an unstable regime, while balanced resource allocation may support a more stable development path.

The identification of equilibrium points and their stability characteristics has practical significance for project planning. A stable equilibrium state may be interpreted as a situation in which the relationship between costs and revenues remains controlled and predictable. In contrast, an unstable node or saddle point may indicate the risk of uncontrolled cost growth,

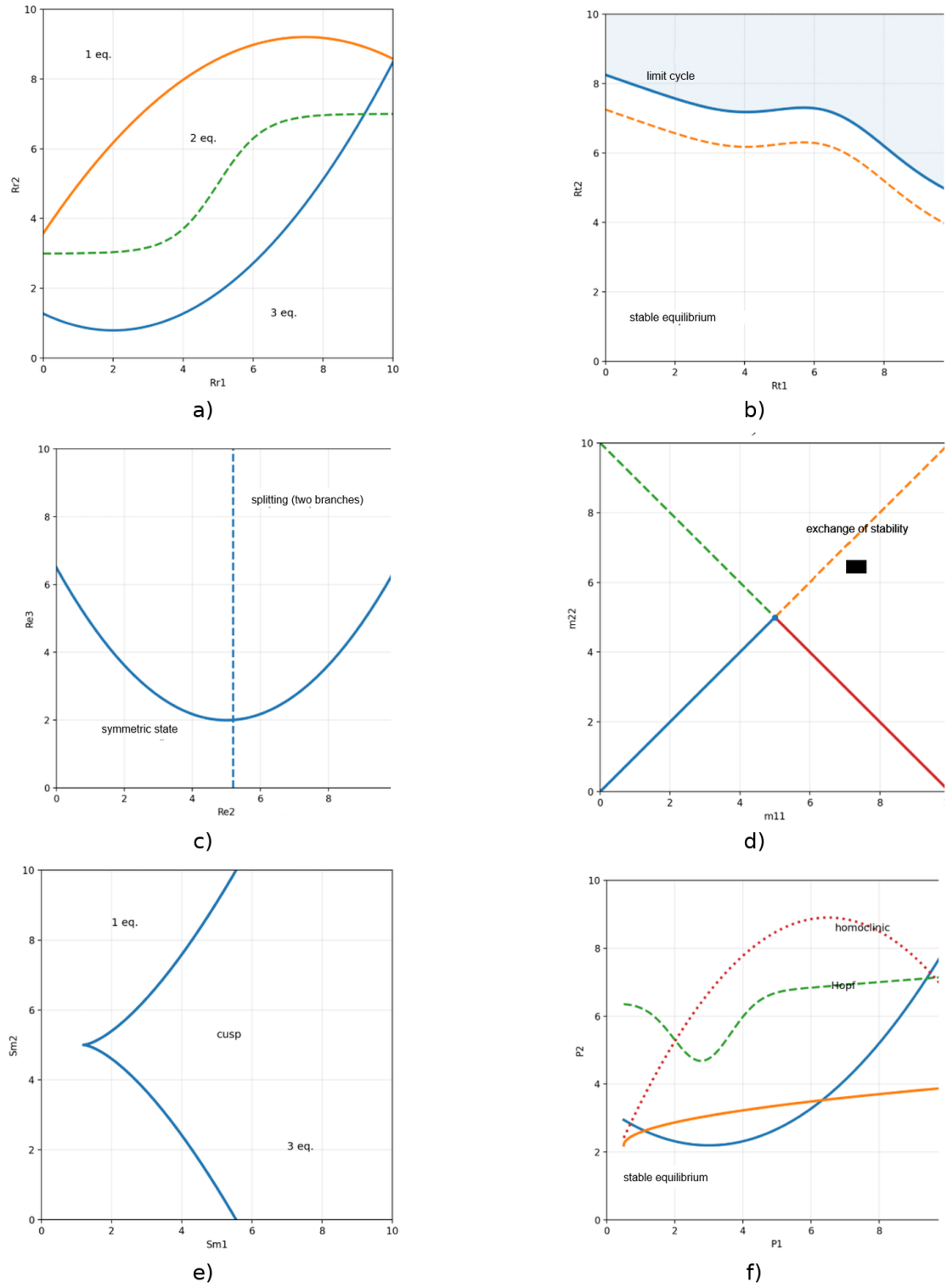


Figure 6. Bifurcation diagram of system (1) with constraints (2), with parameters specified in the section "Parameter identification, in the parameter spaces: a) $Rr_1 : 0 : Rr_2$, b) $Rt_1 : 0 : Rt_2$, c) $Re_2 : 0 : Re_3$, d) $m_{11} : 0 : m_{22}$, e) $Sm_1 : 0 : Sm_2$, f) $P_1 : 0 : P_2$

inefficient resource use, or an imbalance between project expenses and expected revenues. Therefore, the proposed model may be used as an analytical tool for early detection of potentially problematic project configurations.

The bifurcation results are especially relevant for decision-making in energy-efficient construction. They show that critical thresholds exist in the parameter space, and crossing these thresholds may lead to a qualitative transformation of the system's behavior. In practical terms, this means that project managers should monitor not only absolute values of resources and costs but also their combinations. The model can help determine which combinations of labor, equipment, energy consumption, and material costs may create risks for project stability.

At the same time, the study has certain limitations. The numerical experiment was based on representative parameter

values rather than data from a specific construction project. Therefore, the results should be interpreted as a theoretical and methodological demonstration of the model's capabilities. Future studies may expand the model by using empirical project data, incorporating additional economic indicators, and comparing modeled trajectories with real construction project outcomes.

Overall, the results confirm that the proposed differential model can serve as a useful instrument for analyzing, forecasting, and optimizing organizational and technological processes in energy-efficient construction. Its main advantage lies in the possibility of identifying stability conditions and bifurcation transitions that are not visible within traditional static modeling approaches.

4 Conclusions

The study developed and examined a system-integrated organizational and technological differential model for energy-efficient construction. The proposed model makes it possible to describe the dynamics of costs and revenues during the implementation of construction works while taking into account the influence of key resource groups, including labor, equipment, energy, and materials. Unlike static cost estimation approaches, the differential model allows the construction process to be analyzed as a dynamic system whose behavior changes over time depending on initial conditions and combinations of parameters.

The results of the qualitative analysis showed that the proposed system of autonomous differential equations can be used to identify equilibrium points, determine their stability properties, and analyze possible behavioral regimes of the construction project. The application of Lyapunov stability methods made it possible to establish that the equilibrium point of the system may have different characteristics depending on the values of the determining parameter combinations. In particular, the system may demonstrate unstable node, saddle, or saddle-node behavior, which has practical importance for understanding risks in project implementation.

The phase-space analysis and bifurcation diagrams confirmed that the dynamics of the system are influenced not only by individual parameters but also by their combinations. This means that project stability depends on the balanced interaction between labor costs, equipment use, energy consumption, material costs, and expected revenues. Therefore, even small changes in resource allocation may lead to significant transformations in the economic behavior of the construction process.

From a practical perspective, the proposed model can be used as an analytical tool for planning and managing energy-efficient construction projects. It may help project managers evaluate different scenarios, identify potentially unstable configurations, and select more effective strategies for resource distribution. The model also provides a methodological basis for forecasting cost-revenue dynamics and supporting managerial decisions aimed at improving project efficiency.

At the same time, the study has certain limitations. The numerical experiment was based on representative parameter values rather than empirical data from a specific construction project. Therefore, future research should focus on testing the model using real project data, expanding the set of economic and technological indicators, and developing software tools for practical implementation. Further improvement of the model may also include stochastic factors, time delays, environmental indicators, and uncertainty in resource availability.

Overall, the proposed differential model contributes to the development of mathematical and organizational tools for energy-efficient construction management. It provides a deeper understanding of nonlinear project dynamics and creates a foundation for optimizing construction processes under resource constraints.

Author contributions

All authors contributed equally and reviewed and approved the final version of the manuscript.

Conflict of interest

The authors declare that they have no conflicts of interest.

Data availability statement

No empirical dataset was generated or analyzed in this study. The numerical values used in the computational experiments are representative model parameters and are reported within the manuscript. Additional computational details may be obtained from the corresponding author upon reasonable request.

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